

Adaptive Mode Selection in Bidirectional Buffer-aided Relay Networks with Fixed Transmit Powers

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Abstract

We consider a bidirectional network in which two users exchange information with the help of a buffer-aided relay. In such a network without direct link between user 1 and user 2, there exist six possible transmission modes, i.e., four point-to-point modes (user 1-to-relay, user 2-to-relay, relay-to-user 1, relay-to-user 2), a multiple access mode (both users to the relay), and a broadcast mode (the relay to both users). Because of the buffering capability at the relay, the transmissions in the network are not restricted to adhere to a predefined schedule, and therefore, all the transmission modes in the bidirectional relay network can be used adaptively based on the instantaneous channel state information (CSI) of the involved links. For the considered network, assuming fixed transmit powers for both the users and the relay, we derive the optimal transmission mode selection policy which maximizes the sum rate. The proposed policy selects one out of the six possible transmission modes in each time slot based on the instantaneous CSI. Simulation results confirm the effectiveness of the proposed protocol compared to existing protocols.

I. INTRODUCTION

In a bidirectional relay network, two users share information via a relay node [1]. Several protocols have been proposed for such a network under the practical half-duplex constraint, i.e., a node cannot transmit and receive at the same time and in the same frequency band. The simplest protocol is the traditional two-way protocol in which the transmission is accomplished in four successive point-to-point phases: user 1-to-relay, relay-to-user 2, user 2-to-relay, and relay-to-user 1. In contrast, the time division broadcast (TDBC) protocol exploits the broadcasting capability of the wireless medium and combines the relay-to-user 1 and relay-to-user 2 phases into one phase, the broadcast phase [2]. Thereby, the relay broadcasts a superimposed codeword, carrying information for both user 1 and user 2, such that each user is able to recover its intended information by self-interference cancellation. To further increase the spectral efficiency, the multiple access broadcast (MABC) protocol was proposed in which the user 1-to-relay and user 2-to-relay phases are also combined into one phase, the multiple-access phase [3]. In the multiple-access phase, user 1 and user 2 simultaneously transmit to the relay, which decodes both messages. For the bidirectional relay network without a direct link between user 1 and user 2, the capacity regions of all six possible



Fig. 1. Three-node bidirectional relay network consisting of two users and a relay.

transmission modes, i.e., the four point-to-point modes, the multiple access mode, and the broadcast mode, are known [4], [5]. Using this knowledge, a significant research effort has been dedicated to obtaining the achievable rate region and the capacity of the bidirectional relay network with and without fading [1]- [8]. The average capacity of the bidirectional relay network without a direct link between user 1 and user 2 has been obtained for additive white Gaussian noise (AWGN) channels without fading, whereas when fading is present, only achievable rates are known. The reason for this is the fixed schedule of using the transmission modes which is adopted in all existing protocols, and does not exploit the instantaneous channel state information (CSI) of the involved links. For one-way relaying, an adaptive link selection protocol was proposed in [9] where based on the instantaneous CSI, in each time slot, either the source-relay link or the relay-destination link is selected for transmission. To this end, the relay has to have a buffer for data storage. This strategy was shown to achieve the capacity of the one-way relay channel with fading [10].

Motivated by the protocols in [9] and [10], our goal is to utilize all available degrees of freedom in the three-node half-duplex bidirectional relay network with fading, via a buffer-aided and adaptive mode selection protocol. In particular, given the fixed transmit powers of all three nodes in the bidirectional relay network, we find a policy which selects the optimal transmission mode from the six available modes in each time slot such that the sum rate is maximized. A similar problem was considered in [8]. However, the selection policy in [8] does not use all possible modes, i.e., it only selects between two point-to-point modes and the broadcast mode, and assumes that the transmit powers of all three nodes are identical. We will show that considering only the three modes in [8] is not optimal. In fact, the multiple access mode is selected with non-zero probability in the proposed optimal selection policy. Finally, we note that the advantages of buffering come at the expense of an increased end-to-end delay. Therefore, the proposed protocol is suitable for applications that are not sensitive to delay. The delay analysis of the proposed protocol is beyond the scope of the current work and is left for future research.

The remainder of the paper is organized as follows. In Section II, the system model is presented. In Section III, the average sum rate is investigated and the optimal mode selection policy is provided. Simulation results are presented in Section IV. Finally, Section V concludes the paper.

II. SYSTEM MODEL

In this section, we first describe the channel model and review the achievable rates for the six possible transmission modes.

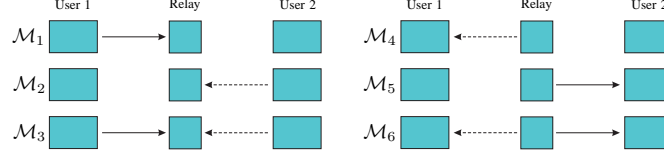


Fig. 2. The six possible transmission modes in the considered bidirectional relay network.

A. Channel Model

We consider a simple network in which users 1 and 2 exchange information with the help of a relay node, as shown in Fig. 1. We assume that there is no direct link between user 1 and user 2, and thus, user 1 and user 2 communicate with each other only through the relay node. We assume that all three nodes in the network are half-duplex. Furthermore, we assume that time is divided into slots of equal length and each node transmits codewords which span one time slot except in the multiple access mode in which the nodes may transmit codewords which span a fraction of the time slot. The duration of one time slot is normalized to one. We assume that the users-to-relay and relay-to-users channels are all impaired by AWGN having zero mean and unit variance, and block fading, i.e., the channel coefficients are constant during one time slot and change from one time slot to the next. Moreover, the channel coefficients are assumed to be reciprocal such that the user 1-to-relay and user 2-to-relay channels are identical to the relay-to-user 1 and relay-to-user 2 channels in each time slot, respectively. Let $h_1(i)$ and $h_2(i)$ denote the channel coefficients between user 1 and the relay and between user 2 and the relay in the i -th time slot, respectively. Furthermore, let $S_1(i) = |h_1(i)|^2$ and $S_2(i) = |h_2(i)|^2$ denote the squares of the channel coefficient amplitudes in the i -th time slot. $S_1(i)$ and $S_2(i)$ are assumed to be ergodic and stationary random processes with means $\Omega_1 = E\{S_1\}$ and $\Omega_2 = E\{S_2\}$ ¹, respectively, where $E\{\cdot\}$ denotes expectation. Since the noise is AWGN, in order to achieve the capacity in each mode, each node has to transmit codewords which are Gaussian distributed. Therefore, the transmitted codewords of user 1, user 2, and the relay are composed of symbols which are modeled as Gaussian random variables with variances P_1, P_2 , and P_r , respectively, where P_j is the transmit power of node $j \in \{1, 2, r\}$. For ease of notation, we define $C(x) \triangleq \log_2(1 + x)$. In the following, we introduce the transmission modes and their achievable rates.

B. Transmission Modes and Their Achievable Rates

In the considered bidirectional relay network only six transmission modes are possible, cf. Fig. 2. The six possible transmission modes are denoted by $\mathcal{M}_1, \dots, \mathcal{M}_6$, and $R_{jj'}(i) \geq 0$, $j, j' \in \{1, 2, r\}$, denotes the transmission rate from node j to node j' in the i -th time slot. Let B_1 and B_2 denote two infinite-size buffers at the relay in which the received information from user 1 and user 2 is stored, respectively. Moreover, $Q_j(i)$, $j \in \{1, 2\}$ denotes as the amount of normalized information in bits/symbol available in buffer B_j in the i -th time slot. Using this notation, the transmission modes and their respective rates are presented in the following:

¹In the paper, we drop time index i in expectations for notational simplicity.

\mathcal{M}_1 : User 1 transmits to the relay and user 2 is silent. In this mode, the maximum rate from user 1 to the relay in the i -th time slot is given by $R_{1r}(i) = C_{1r}(i)$, where $C_{1r}(i) = C(P_1 S_1(i))$. The relay decodes this information and stores it in buffer B_1 . Therefore, the amount of information in buffer B_1 increases to $Q_1(i) = Q_1(i-1) + R_{1r}(i)$.

\mathcal{M}_2 : User 2 transmits to the relay and user 1 is silent. In this mode, the maximum rate from user 2 to the relay in the i -th time slot is given by $R_{2r}(i) = C_{2r}(i)$, where $C_{2r}(i) = C(P_2 S_2(i))$. The relay decodes this information and stores it in buffer B_2 . Therefore, the amount of information in buffer B_2 increases to $Q_2(i) = Q_2(i-1) + R_{2r}(i)$.

\mathcal{M}_3 : Both users 1 and 2 transmit to the relay simultaneously. For this mode, we assume that multiple access transmission is used, see [5]. Thereby, the maximum achievable sum rate in the i -th time slot is given by $R_{1r}(i) + R_{2r}(i) = C_r(i)$, where $C_r(i) = C(P_1 S_1(i) + P_2 S_2(i))$. Since user 1 and user 2 transmit independent messages, the sum rate, $C_r(i)$, can be decomposed into two rates, one from user 1 to the relay and the other one from user 2 to the relay. Moreover, these two capacity rates can be achieved via time sharing and successive interference cancellation. Thereby, in the first $0 \leq t(i) \leq 1$ fraction of the i -th time slot, the relay first decodes the codeword received from user 2 and considers the signal from user 1 as noise. Then, the relay subtracts the signal component of user 2 from the received signal and decodes the codeword received from user 1. A similar procedure is performed in the remaining $1 - t(i)$ fraction of the i -th time slot but now the relay first decodes the codeword from user 1 and treats the signal of user 2 as noise, and then decodes the codeword received from user 2. Therefore, for a given $t(i)$, we decompose $C_r(i)$ as $C_r(i) = C_{12r}(i) + C_{21r}(i)$ and the maximum rates from users 1 and 2 to the relay in the i -th time slot are $R_{1r}(i) = C_{12r}(i)$ and $R_{2r}(i) = C_{21r}(i)$, respectively. $C_{12r}(i)$ and $C_{21r}(i)$ are given by

$$C_{12r}(i) = t(i)C(P_1 S_1(i)) + (1 - t(i))C\left(\frac{P_1 S_1(i)}{1 + P_2 S_2(i)}\right) \quad (1a)$$

$$C_{21r}(i) = (1 - t(i))C(P_2 S_2(i)) + t(i)C\left(\frac{P_2 S_2(i)}{1 + P_1 S_1(i)}\right) \quad (1b)$$

The relay decodes the information received from user 1 and user 2 and stores it in its buffers B_1 and B_2 , respectively. Therefore, the amount of information in buffers B_1 and B_2 increase to $Q_1(i) = Q_1(i-1) + R_{1r}(i)$ and $Q_2(i) = Q_2(i-1) + R_{2r}(i)$, respectively.

\mathcal{M}_4 : The relay transmits the information received from user 2 to user 1. Specifically, the relay extracts the information from buffer B_2 , encodes it into a codeword, and transmits it to user 1. Therefore, the transmission rate from the relay to user 1 in the i -th time slot is limited by both the capacity of the relay-to-user 1 channel and the amount of information stored in buffer B_2 . Thus, the maximum transmission rate from the relay to user 1 is given by $R_{r1}(i) = \min\{C_{r1}(i), Q_2(i-1)\}$, where $C_{r1}(i) = C(P_r S_1(i))$. Therefore, the amount of information in buffer B_2 decreases to $Q_2(i) = Q_2(i-1) - R_{r1}(i)$.

\mathcal{M}_5 : This mode is identical to \mathcal{M}_4 with user 1 and 2 switching places. The maximum transmission rate from the relay to user 2 is given by $R_{r2}(i) = \min\{C_{r2}(i), Q_1(i-1)\}$, where $C_{r2}(i) = C(P_r S_2(i))$ and the amount of information in buffer B_1 decreases to $Q_1(i) = Q_1(i-1) - R_{r2}(i)$.

\mathcal{M}_6 : The relay broadcasts to both user 1 and user 2 the information received from user 2 and user 1, respectively. Specifically, the relay extracts the information intended for user 2 from buffer B_1 and the information intended for

user 1 from buffer B_2 . Then, based on the scheme in [4], it constructs a superimposed codeword which contains the information from both users and broadcasts it to both users. Thus, in the i -th time slot, the maximum rates from the relay to users 1 and 2 are given by $R_{r1}(i) = \min\{C_{r1}(i), Q_2(i-1)\}$ and $R_{r2}(i) = \min\{C_{r2}(i), Q_1(i-1)\}$, respectively. Therefore, the amount of information in buffer B_1 and B_2 decrease to $Q_1(i) = Q_1(i-1) - R_{r2}(i)$ and $Q_2(i) = Q_2(i-1) - R_{r1}(i)$, respectively.

Our aim is to develop an optimal selection policy which selects one of the six possible transmission modes in each time slot such that, given P_1 , P_2 , and P_r , the average sum rate of both users is maximized. To this end, we introduce six binary variables $q_k(i) \in \{0, 1\}$, $k = 1, \dots, 6$, where $q_k(i)$ indicates whether or not transmission mode \mathcal{M}_k is selected in the i -th time slot. In particular, $q_k(i) = 1$ if mode \mathcal{M}_k is selected and $q_k(i) = 0$ if it is not selected in the i -th time slot. Moreover, since in each time slot only one of the six transmission modes can be selected, only one of the mode selection variables is equal to one and the others are zero, i.e., $\sum_{k=1}^6 q_k(i) = 1$ holds.

In the proposed framework, we assume that one node (e.g. the relay node) is responsible for deciding which transmission mode is selected based on the full CSI of both links and the proposed protocol, cf. Theorem 2. Then, it sends its decision to the other nodes and transmission in time slot i begins. Furthermore, for each node to be able to decode their intended messages and adapt their transmission rates, all three nodes have to have full CSI of both links.

III. ADAPTIVE MODE SELECTION

In this section, we first investigate the achievable average sum rate of the network. Then, we formulate a maximization problem whose solution yields the maximum average sum rate.

A. Achievable Average Sum Rate

We assume that user 1 and user 2 always have enough information to send in all time slots and that the number of time slots, N , satisfies $N \rightarrow \infty$. Therefore, using $q_k(i)$, the user 1-to-relay, user 2-to-relay, relay-to-user 1, and relay-to-user 2 average transmission rates, denoted by \bar{R}_{1r} , \bar{R}_{2r} , \bar{R}_{r1} , and \bar{R}_{r2} , respectively, are obtained as

$$\bar{R}_{1r} = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N [q_1(i)C_{1r}(i) + q_3(i)C_{12r}(i)] \quad (2a)$$

$$\bar{R}_{2r} = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N [q_2(i)C_{2r}(i) + q_3(i)C_{21r}(i)] \quad (2b)$$

$$\bar{R}_{r1} = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N [q_4(i) + q_6(i)] \min\{C_{r1}(i), Q_2(i-1)\} \quad (2c)$$

$$\bar{R}_{r2} = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N [q_5(i) + q_6(i)] \min\{C_{r2}(i), Q_1(i-1)\}. \quad (2d)$$

The average rate from user 1 to user 2 is the average rate that user 2 receives from the relay, i.e., \bar{R}_{r2} . Similarly, the average rate from user 2 to user 1 is the average rate that user 1 receives from the relay, i.e., \bar{R}_{r1} . In the following

theorem, we introduce a useful condition for the queues in the buffers of the relay leading to the optimal mode selection policy.

Theorem 1 (Optimal Queue Condition): The maximum sum rate, $\bar{R}_{r1} + \bar{R}_{r2}$, for the considered bidirectional relay network is obtained when the queues in buffers B_1 and B_2 are at the edge of non-absorption. More precisely, the following conditions must hold for the maximum sum rate:

$$\bar{R}_{1r} = \bar{R}_{r2} = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N [q_5(i) + q_6(i)] C_{r2}(i) \quad (3a)$$

$$\bar{R}_{2r} = \bar{R}_{r1} = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N [q_4(i) + q_6(i)] C_{r1}(i) \quad (3b)$$

where \bar{R}_{1r} and \bar{R}_{2r} are given by (2a) and (2b), respectively.

Proof Please refer to Appendix A.

Using this theorem, we now derive the optimal transmission mode selection policy.

B. Optimal Mode Selection Protocol

The available degrees of freedom in the considered network are the mode selection variables $q_k(i)$, $\forall k, i$ and the time sharing variable in the multiple access mode, $t(i)$, $\forall i$. In the following, we formulate a problem for optimization of $q_k(i)$, $\forall k, i$ and $t(i)$, $\forall i$ such that the average sum rate, $\bar{R}_{r1} + \bar{R}_{r2}$, is maximized:

$$\begin{aligned} & \underset{q_k(i), t(i), \forall i, k}{\text{maximize}} && \bar{R}_{1r} + \bar{R}_{2r} \\ & \text{subject to} && \text{C1 : } \bar{R}_{1r} = \bar{R}_{r2} \\ & && \text{C2 : } \bar{R}_{2r} = \bar{R}_{r1} \\ & && \text{C3 : } \sum_{k=1}^6 q_k(i) = 1, \forall i \\ & && \text{C4 : } q_k(i)[1 - q_k(i)] = 0, \forall i, k \\ & && \text{C5 : } 0 \leq t(i) \leq 1, \forall i. \end{aligned} \quad (4)$$

where C1 and C2 are due to Theorem 1, C3 and C4 guarantee that only one of the transmission modes is selected in each time slot, and C5 specifies the interval of the time sharing variable. Considering constraints C1 and C2, the sum rate, $\bar{R}_{r1} + \bar{R}_{r2}$, is also given by $\bar{R}_{1r} + \bar{R}_{2r}$.

As will be seen later, the solution of (4), i.e., the optimal mode selection policy may require coin flips. Therefore, we define $X_n(i) \in \{0, 1\}$ as the outcomes of the n -th coin flip in the i -th time slot. The probabilities of the possible outcomes of the n -th coin flip are $\Pr\{X_n(i) = 1\} = p_n$ and $\Pr\{X_n(i) = 0\} = 1 - p_n$. In the following theorem, we provide the solution to the maximization problem in (4). To this end, we define three mutually exclusive signal-to-noise ratio (SNR) regions and each region requires a different optimal selection policy. For given Ω_1 , Ω_2 , P_1 , P_2 , and P_r , exactly one of the SNR regions is applicable.

Theorem 2 (Mode Selection Policy): The optimal mode selection policy which maximizes the average sum rate of the considered three-node half-duplex bidirectional relay network with AWGN and block fading is given by

$$q_{k^*}(i) = \begin{cases} 1, & k^* = \arg \max_{k=1,\dots,6} \{\mathcal{I}_k(i) \Lambda_k(i)\} \\ 0, & \text{otherwise} \end{cases} \quad (5)$$

where $\Lambda_k(i)$ is referred to as the selection metric, given by

$$\Lambda_1(i) = (1 - \mu_1)C_{1r}(i) \quad (6a)$$

$$\Lambda_2(i) = (1 - \mu_2)C_{2r}(i) \quad (6b)$$

$$\Lambda_3(i) = (1 - \mu_1)C_{12r}(i) + (1 - \mu_2)C_{21r}(i) \quad (6c)$$

$$\Lambda_4(i) = \mu_2 C_{r1}(i) \quad (6d)$$

$$\Lambda_5(i) = \mu_1 C_{r2}(i) \quad (6e)$$

$$\Lambda_6(i) = \mu_1 C_{r2}(i) + \mu_2 C_{r1}(i) \quad (6f)$$

and $\mathcal{I}_k(i) \in \{0, 1\}$ is a binary indicator variable which determines whether \mathcal{M}_k is a *possible candidate* for selection in the i -th time slot, i.e., mode \mathcal{M}_k cannot be selected if $\mathcal{I}_k(i) = 0$, and μ_1 and μ_2 are decision thresholds. For the values of $\mathcal{I}_k(i)$, the optimal values of μ_1 and μ_2 in (6), and the optimal value of $t(i)$ in $C_{12r}(i)$ and $C_{21r}(i)$ in (6c), we distinguish three mutually exclusive SNR regions:

SNR Region \mathcal{R}_1 : SNR region \mathcal{R}_1 is characterized by $\{P_2 > P_r \text{ AND } \omega_1 < 1\}$ OR $\{P_2 < P_r \text{ AND } \omega_2 < 1\}$ OR $\{P_2 = P_r \text{ AND } \omega_3^l < \omega_3^u\}$ with $\omega_1 = \frac{E\{qC_{r2}\}}{E\{(1-q)[C_r - C_{2r}]\}}, \omega_2 = \frac{E\{qC_{2r}\}}{E\{(1-q)C_{r1}\}}, \omega_3^l = \frac{E\{C_{r2}\}}{E\{C_r\}},$ and $\omega_3^u = \frac{E\{C_{r1}\}}{E\{C_{r1} + C_{2r}\}},$ where $q(i)$ is defined as

$$q(i) = \begin{cases} 1, & \text{if } \left\{ P_2 > P_r \text{ AND } \Lambda_3 \leq \Lambda_6 \Big|_{\substack{t(i)=0, \forall i \\ \mu_1=1, \mu_2=\mu_2^*}} \right\} \\ & \text{OR } \left\{ P_2 < P_r \text{ AND } \Lambda_3 \geq \Lambda_6 \Big|_{\substack{t(i)=0, \forall i \\ \mu_1=\mu_1^*, \mu_2=0}} \right\} \\ 0, & \text{otherwise} \end{cases} \quad (7)$$

In (7), if $P_2 > P_r$, μ_2^* is chosen such that $E\{(1-q)C_{2r}\} = E\{qC_{r1}\}$ holds, whereas if $P_2 < P_r$, μ_1^* is chosen such that $E\{q[C_r - C_{2r}]\} = E\{(1-q)C_{r2}\}$ holds.

For this SNR region, $t(i) = 0, \forall i$, and $\mathcal{I}_k(i) = 0$, for $k = 1, 4, \forall i$, whereas $\mathcal{I}_k(i)$ for $k = 2, 3, 5, 6$, and the thresholds μ_1 and μ_2 are as follows

$$\text{If } P_2 > P_r \text{ then } \begin{cases} \mu_1 = 1 \\ \mu_2 = \mu_2^* \end{cases} \text{ and } \begin{cases} \mathcal{I}_5(i) = 0 \\ \mathcal{I}_3(i) = 1 - \mathcal{I}_2(i) = X_1(i) \\ \mathcal{I}_6(i) = 1 \end{cases}$$

$$\begin{aligned}
& \text{If } P_2 < P_r \text{ then } \begin{cases} \mu_1 = \mu_1^* \\ \mu_2 = 0 \end{cases} \text{ and } \begin{cases} \mathcal{I}_2(i) = 0, \\ \mathcal{I}_6(i) = 1 - \mathcal{I}_5(i) = X_2(i) \\ \mathcal{I}_3(i) = 1 \end{cases} \\
& \text{If } P_2 = P_r \text{ then } \begin{cases} \mu_1 = 1 \\ \mu_2 = 0 \end{cases} \text{ and } \begin{cases} \mathcal{I}_2(i) = X_3(i)[1 - X_4(i)] \\ \mathcal{I}_3(i) = X_3(i)X_4(i) \\ \mathcal{I}_5(i) = [1 - X_3(i)][1 - X_5(i)] \\ \mathcal{I}_6(i) = [1 - X_3(i)]X_5(i) \end{cases}
\end{aligned}$$

where the coin flip probabilities are $p_1 = \omega_1, p_2 = \omega_2, p_4 = \frac{(1-p_3)\omega_3^l}{p_3(1-\omega_3^l)}, p_5 = \frac{p_3(1-\omega_3^u)}{(1-p_3)\omega_3^u}$, and for p_3 , we have the freedom to choose any value in the interval $[\omega_3^l, \omega_3^u]$.

SNR Region \mathcal{R}_2 : SNR region \mathcal{R}_2 is characterized by $\{P_1 > P_r \text{ AND } \omega_1 < 1\}$ OR $\{P_1 < P_r \text{ AND } \omega_2 < 1\}$ OR $\{P_1 = P_r \text{ AND } \omega_3^l < \omega_3^u\}$ with $\omega_1 = \frac{E\{qC_{r1}\}}{E\{(1-q)[C_r - C_{1r}]\}}, \omega_2 = \frac{E\{qC_{1r}\}}{E\{(1-q)C_{r2}\}}, \omega_3^l = \frac{E\{C_{r1}\}}{E\{C_r\}},$ and $\omega_3^u = \frac{E\{C_{r2}\}}{E\{C_{r2} + C_{1r}\}},$ where $q(i)$ is defined as

$$q(i) = \begin{cases} 1, & \text{if } \left\{ P_1 > P_r \text{ AND } [\Lambda_3 \leq \Lambda_6]_{\mu_1 = \mu_1^*, \mu_2 = 1}^{t(i)=1, \forall i} \right\} \\ & \text{OR } \left\{ P_1 < P_r \text{ AND } [\Lambda_3 \geq \Lambda_6]_{\mu_1 = 0, \mu_2 = \mu_2^*}^{t(i)=1, \forall i} \right\} \\ 0, & \text{otherwise} \end{cases} \quad (8)$$

In (8), if $P_1 > P_r$, the threshold μ_1^* is chosen such that $E\{(1-q)C_{1r}\} = E\{qC_{r2}\}$ holds, whereas if $P_1 < P_r$, the threshold μ_2^* is chosen such that $E\{q[C_r < C_{1r}]\} = E\{(1-q)C_{r1}\}$ holds.

For this SNR region, $t(i) = 1, \forall i$, and $\mathcal{I}_k(i) = 0$, for $k = 2, 5, \forall i$, whereas $\mathcal{I}_k(i)$ for $k = 1, 3, 4, 6$, and the thresholds μ_1 and μ_2 are as follows

$$\begin{aligned}
& \text{If } P_1 > P_r \text{ then } \begin{cases} \mu_1 = \mu_1^* \\ \mu_2 = 1 \end{cases} \text{ and } \begin{cases} \mathcal{I}_4(i) = 0 \\ \mathcal{I}_3(i) = 1 - \mathcal{I}_1(i) = X_1(i) \\ \mathcal{I}_6(i) = 1 \end{cases} \\
& \text{If } P_1 < P_r \text{ then } \begin{cases} \mu_1 = 0 \\ \mu_2 = \mu_2^* \end{cases} \text{ and } \begin{cases} \mathcal{I}_1(i) = 0, \\ \mathcal{I}_6(i) = 1 - \mathcal{I}_4(i) = X_2(i) \\ \mathcal{I}_3(i) = 1 \end{cases} \\
& \text{If } P_1 = P_r \text{ then } \begin{cases} \mu_1 = 0 \\ \mu_2 = 1 \end{cases} \text{ and } \begin{cases} \mathcal{I}_1(i) = X_3(i)[1 - X_4(i)] \\ \mathcal{I}_3(i) = X_3(i)X_4(i) \\ \mathcal{I}_4(i) = [1 - X_3(i)][1 - X_5(i)] \\ \mathcal{I}_6(i) = [1 - X_3(i)]X_5(i) \end{cases}
\end{aligned}$$

where the coin flip probabilities are $p_1 = \omega_1, p_2 = \omega_2, p_4 = \frac{(1-p_3)\omega_3^l}{p_3(1-\omega_3^l)}, p_5 = \frac{p_3(1-\omega_3^u)}{(1-p_3)\omega_3^u}$, and for p_3 , we have the freedom to choose any value in the interval $[\omega_3^l, \omega_3^u]$.

SNR Region \mathcal{R}_0 : The SNR region is \mathcal{R}_0 if and only if it is not \mathcal{R}_1 or \mathcal{R}_2 . For SNR region \mathcal{R}_0 , $\mathcal{I}_k(i) = 0$ for $k = 1, 2, 4, 5$ and $\mathcal{I}_k(i) = 1$ for $k = 3, 6, \forall i$, and $t(i)$ is given by

$$t(i) = \begin{cases} 0, & \text{if } t \leq 0 \\ t, & \text{if } 0 < t < 1 \\ 1, & \text{if } t \geq 1 \end{cases} \quad (9)$$

where $t = \frac{E\{q(C_r - C_{2r})\} - E\{(1-q)C_{r2}\}}{E\{q(C_r - C_{1r} - C_{2r})\}}$ and $q(i)$ is given by

$$q(i) = \begin{cases} 1, & \text{if } \frac{C_r(i)}{C_{r1}(i) + C_{r2}(i)} \geq \frac{\mu^*}{\mu^* - 1} \\ 0, & \text{if otherwise} \end{cases} \quad (10)$$

where μ^* is chosen such that $E\{qC_r\} = E\{(1-q)(C_{r1} + C_{r2})\}$ holds. Moreover, if $0 < t < 1$, we obtain $\mu_1 = \mu_2 = \mu^*$. Otherwise, if $t \leq 0$ or $t \geq 1$, we obtain $\mu_1 \geq \mu_2$ and $\mu_1 \leq \mu_2$, respectively, and μ_1 and μ_2 are chosen such that C1 and C2 in (4) hold.

Proof Please refer to Appendix B.

We note that in the proposed optimal policy, regardless of the SNR region, the multiple access and broadcast modes are always selected with non-zero probability. Only in the extreme case, when the average SNR of one of the links is much larger than the average SNR of the other link, some of the point-to-point modes are also selected in the optimal policy. Also, the mode selection metric $\Lambda_k(i)$ includes two parts. The first part is the capacity of mode \mathcal{M}_k , and the second part are the decision thresholds μ_1 and/or μ_2 . The capacity is a function of the instantaneous CSI, but the decision thresholds μ_1 and μ_2 depend only on the statistics of the channels. Hence, the decision thresholds, μ_1 and μ_2 , can be obtained offline and used as long as the channel statistics remain unchanged.

Remark: To find the optimal values of μ_1 and μ_2 , we need one/two-dimensional searches over $[0, 1]$. Specifically, in SNR regions \mathcal{R}_1 and \mathcal{R}_2 , at least one of the thresholds μ_1 and μ_2 is given. Therefore, at most one one-dimensional search is required. In SNR region \mathcal{R}_0 , if $0 \leq \omega \leq 1$, we obtain $\mu_1 = \mu_2 = \mu^*$, therefore, a one-dimensional search is required to determine μ^* . However, if $\omega < 0$ or $\omega > 1$, we need a two-dimensional search to find μ_1 and μ_2 .

IV. SIMULATION RESULTS

In this section, we evaluate the maximum average sum rate in the considered bidirectional relay network for Rayleigh fading. Thus, channel gains $S_1(i)$ and $S_2(i)$ follow exponential distributions with averages Ω_1 and Ω_2 , respectively. Furthermore, the transmit powers of user 1 and user 2 are identical and set to $P_1 = P_2 = 10$ dB, and $\Omega_2 = 0$ dB. The number of time slots used in the simulations is $N = 10^6$.

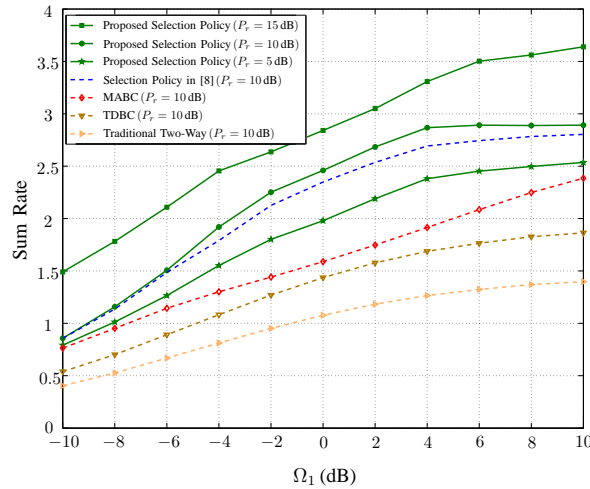


Fig. 3. Maximum achievable sum rate versus Ω_1 .

Fig. 3 illustrates the maximum achievable sum rate versus Ω_1 . In order to simulate all possible cases introduced in Theorem 2, we consider $P_r = 5, 10, 15$ dB, which yields the solid lines in Fig. 3. As the value of Ω_1 varies from -10 dB to 10 dB in Fig. 3, the SNR region changes from \mathcal{R}_2 to \mathcal{R}_0 and then from \mathcal{R}_0 to \mathcal{R}_1 . Since in SNR region \mathcal{R}_2 the bottleneck is the relay-to-user 1 link, reducing Ω_1 reduces the sum rate. On the other hand, in SNR region \mathcal{R}_1 , the bottleneck link is the relay-to-user 2 link, and therefore, for large Ω_1 , the maximum average sum rate saturates.

As performance benchmarks, we consider the traditional two-way, the TDBC [2], the MABC [3], and the buffer-aided protocol in [8]. Since in [8], the protocol is derived for $P_1 = P_2 = P_r$, in Fig. 3, we only show the sum rates of the benchmark schemes for $P_r = 10$ dB. From the comparison in Fig. 3, we observe that a considerable gain is obtained by adaptive mode selection compared to the traditional two-way, TDBC, and MABC protocols. Furthermore, a small gain is obtained compared to the buffer-aided protocol in [8] which selects only the user 1-to-relay, user 2-to-relay, and broadcast modes adaptively. Finally, we note that our protocol is derived for given Ω_1 , Ω_2 , P_1 , P_2 , and P_r , and without a constraint on the total average power consumed by all nodes. The problem with a total average power constraint would lead to a different solution.

V. CONCLUSION

We have derived the optimal transmission mode selection policy for sum rate maximization of the three-node half-duplex bidirectional buffer-aided relay network with fading links and fixed transmit powers at the nodes. The proposed selection policy determines the optimal transmission mode based on the instantaneous CSI of the involved links in each time slot and their long-term statistics. Simulation results confirmed that the proposed selection policy outperforms the existing protocols from the literature in terms of average sum rate.

APPENDIX A

PROOF OF THEOREM 1 (OPTIMAL QUEUE CONDITION)

Let \bar{C}_{r1} and \bar{C}_{r2} denote the average rates achieved by the relay assuming that buffers B_1 and B_2 always have enough information to supply. Then, \bar{C}_{r1} and \bar{C}_{r2} are given by

$$\bar{C}_{r1} = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N [q_4(i) + q_6(i)] C_{r1}(i) \quad (11a)$$

$$\bar{C}_{r2} = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N [q_5(i) + q_6(i)] C_{r2}(i). \quad (11b)$$

If we assume that $\bar{R}_{2r} \geq \bar{C}_{r1}$ holds, i.e., the average rate flowing into the buffer is larger than or equal to the average capacity that can flow out of the buffer, then the buffer always has enough information to supply and \bar{R}_{r1} in (2) becomes $\bar{R}_{r1} = \bar{C}_{r1}$, where \bar{C}_{r1} is given in (11a). For a proof of this property, we refer to the proof of Theorem 1 in [9, Appendix A]. On the other hand, if we assume that $\bar{R}_{2r} < \bar{C}_{r1}$ holds, i.e., the average information flowing into the buffer is less than the average capacity that can flow out of the buffer, then by the law of conservation of flow, we obtain $\bar{R}_{r1} = \bar{R}_{2r}$. Similar results hold for \bar{R}_{r2} , \bar{R}_{1r} , and \bar{C}_{r2} . Hence, \bar{R}_{r1} and \bar{R}_{r2} can be written as

$$\bar{R}_{r1} = \begin{cases} \bar{C}_{r1}, & \text{if } \bar{R}_{2r} \geq \bar{C}_{r1} \\ \bar{R}_{2r}, & \text{otherwise} \end{cases} \quad (12a)$$

$$\bar{R}_{r2} = \begin{cases} \bar{C}_{r2}, & \text{if } \bar{R}_{1r} \geq \bar{C}_{r2} \\ \bar{R}_{1r}, & \text{otherwise} \end{cases} \quad (12b)$$

On the other hand, (12) can also be written as

$$\bar{R}_{r1} = \min\{\bar{R}_{2r}, \bar{C}_{r1}\} \quad (13a)$$

$$\bar{R}_{r2} = \min\{\bar{R}_{1r}, \bar{C}_{r2}\} \quad (13b)$$

i.e., the average rate flowing out of buffer B_1 (B_2) is limited by the minimum of the average rate flowing into the buffer and the capacity rate that can flow out of the buffer. We will show that, in the optimal selection policy, $\bar{R}_{1r} = \bar{C}_{r2}$ and $\bar{R}_{2r} = \bar{C}_{r1}$ must hold, and as a result from (13a) and (13b), we obtain $\bar{R}_{1r} = \bar{R}_{r2} = \bar{C}_{r2}$ and $\bar{R}_{2r} = \bar{R}_{r1} = \bar{C}_{r1}$. To show this, we denote by I_k the set comprising the time slots i in which $q_k(i) = 1$, i.e., the elements in I_k represent the time slots in which transmission mode \mathcal{M}_k is selected in the optimal solution. Note that $\sum_{k=1}^6 |I_k| = N$, where $|\cdot|$ denotes the cardinality of a set and \emptyset denotes an empty set with zero cardinality. We will show that, except for the case when both arguments of the min functions in (13a) and (13b) are equal, all other cases result in suboptimal sum rate or can be safely replaced with the case when both arguments of the min function are equal without reducing the sum rate. For the two min functions, we have to consider the following nine cases:

Case 1: If $\bar{R}_{1r} > \bar{C}_{r2}$ and $\bar{R}_{2r} > \bar{C}_{r1}$, then we can move any indices i from I_1, I_2 , or I_3 to I_6 and thereby increase

both \bar{C}_{r1} and \bar{C}_{r2} , and thus improve the sum rate. Hence, $\bar{R}_{1r} > \bar{C}_{r2}$ and $\bar{R}_{2r} > \bar{C}_{r1}$ cannot hold since the resulting sum rate is suboptimal.

Case 2: If $\bar{R}_{1r} = \bar{C}_{r2}$ and $\bar{R}_{2r} > \bar{C}_{r1}$, then if $I_2 \neq \emptyset$, one can move some indices from I_2 to I_4 which maintains $\bar{R}_{1r} = \bar{C}_{r2}$ and increases \bar{C}_{r1} , and consequently increases the sum rate, thereby contradicting optimality. On the other hand, if $I_2 = \emptyset$, we have $I_3 \neq \emptyset$ since otherwise, $\bar{R}_{2r} = 0$ and because we have assumed $\bar{R}_{2r} > \bar{C}_{r1}$, we obtain $\bar{R}_{2r} = \bar{C}_{r1} = 0$. Hence, assume $I_2 = \emptyset$ and $I_3 \neq \emptyset$, then there are two possibilities. The first is when $t(i) = 1, \forall i$ for which $q_3(i) = 1$. Then, since $C_{12r}(i)$ becomes $C_{12r}(i) = C_{1r}(i)$, we can move some indices from I_3 to I_1 , which maintains $\bar{R}_{1r} = \bar{C}_{r2}$ and decreases \bar{R}_{2r} until we obtain $\bar{R}_{2r} = \bar{C}_{r1}$, thereby contradicting $\bar{R}_{2r} > \bar{C}_{r1}$. However, if $t(i) \neq 1$ for some time slots i for which $q_3(i) = 1$, then since $C_{12r}(i) \leq C_{1r}(i)$ holds, we can move some indices from I_3 to I_1 and I_4 such that $\bar{R}_{1r} = \bar{C}_{r2}$ is maintained and \bar{C}_{r1} is increased and thereby the sum rate is increased, which results in a contradiction of the optimal sum rate assumption.

Case 3: If $\bar{R}_{1r} = \bar{C}_{r2}$ and $\bar{R}_{2r} < \bar{C}_{r1}$, then if $I_4 \neq \emptyset$, we can move some indices of I_4 to I_2 which maintains $\bar{R}_{1r} = \bar{C}_{r2}$ and increases \bar{R}_{2r} and consequently increases the sum rate and thus contradicts optimality. On the other hand, if $I_4 = \emptyset$, we must have $I_6 \neq \emptyset$, otherwise, $\bar{C}_{r2} = 0$, and since we have assumed $\bar{R}_{2r} < \bar{C}_{r1}$, we obtain $\bar{R}_{2r} = \bar{C}_{r1} = 0$. However, if $I_6 \neq \emptyset$, we can move some of the indices in I_6 to I_5 which maintains $\bar{R}_{1r} = \bar{C}_{r2}$ and decreases \bar{C}_{r1} until we obtain $\bar{R}_{2r} = \bar{C}_{r1}$, thereby contradicting $\bar{R}_{2r} < \bar{C}_{r1}$.

Case 4: If $\bar{R}_{1r} > \bar{C}_{r2}$ and $\bar{R}_{2r} < \bar{C}_{r1}$, then if $I_4 \neq \emptyset$, we can move some indices of I_4 to I_5 , which increases \bar{C}_{r2} and consequently the sum rate and thereby contradicting optimality. However, if $I_4 = \emptyset$, then we must have $I_6 \neq \emptyset$ otherwise $\bar{C}_{r1} = 0$ leading to $\bar{R}_{2r} = \bar{C}_{r1} = 0$ and the rest of the conclusions are similar to that in Case 2. Then, if $I_6 \neq \emptyset$, one can move some of the indices from I_6 to I_5 which maintains $\bar{R}_{1r} > \bar{C}_{r2}$ but decreases \bar{C}_{r1} until we obtain $\bar{R}_{2r} = \bar{C}_{r1}$ and the rest of conclusions are similar to those in Case 2.

The conclusions for the four remaining cases, $(\bar{R}_{1r} < \bar{C}_{r2} \text{ and } \bar{R}_{2r} < \bar{C}_{r1})$, $(\bar{R}_{1r} > \bar{C}_{r2} \text{ and } \bar{R}_{2r} = \bar{C}_{r1})$, $(\bar{R}_{1r} < \bar{C}_{r2} \text{ and } \bar{R}_{2r} = \bar{C}_{r1})$, and $(\bar{R}_{1r} < \bar{C}_{r2} \text{ and } \bar{R}_{2r} > \bar{C}_{r1})$ are similar to those in Case 1, Case 2, Case 3 and Case 4, respectively. Therefore, the final case remaining for the optimal solution is $\bar{R}_{r2} = \bar{R}_{1r} = \bar{C}_{r2}$ and $\bar{R}_{r1} = \bar{R}_{2r} = \bar{C}_{r1}$. This completes the proof.

APPENDIX B

PROOF OF THEOREM 2 (MODE SELECTION PROTOCOL)

In this appendix, we solve the optimization problem given by (4). We first relax the binary condition for $q_k(i)$, i.e., $q_k(i)[1 - q_k(i)] = 0$, to $0 \leq q_k(i) \leq 1$, and later in Appendix C, we prove that the binary relaxation does not affect the maximum average sum rate. In the following, we investigate the Karush-Kuhn-Tucker (KKT) necessary conditions of the relaxed optimization problem and then show that the necessary conditions result in a unique sum rate and thus the solution is optimal.

To simplify the usage of the KKT conditions, we formulated a minimization problem equivalent to the relaxed

$$\begin{aligned}
\mathcal{L}(q_k(i), t(i), \lambda(i), \alpha_k(i), \beta_k(i), \phi_l(i)) = \\
\text{for } \forall i, k, l \\
-(\bar{R}_{1r} + \bar{R}_{2r}) + \mu_1(\bar{R}_{1r} - \bar{R}_{r2}) + \mu_2(\bar{R}_{2r} - \bar{R}_{r1}) + \sum_{i=1}^N \lambda(i) \left(\sum_{k=1}^6 q_k(i) - 1 \right) \\
+ \sum_{i=1}^N \sum_{k=1}^6 \alpha_k(i) (q_k(i) - 1) - \sum_{i=1}^N \sum_{k=1}^6 \beta_k(i) q_k(i) + \sum_{i=1}^N \phi_1(i)(t(i) - 1) - \sum_{i=1}^N \phi_0(i)t(i)
\end{aligned} \tag{15}$$

maximization problem in (4) as follows

$$\begin{aligned}
& \underset{q_k(i), t(i), \forall i, k}{\text{minimize}} && -(\bar{R}_{1r} + \bar{R}_{2r}) \\
& \text{subject to} && \text{C1: } \bar{R}_{1r} - \bar{R}_{r2} = 0 \\
& && \text{C2: } \bar{R}_{2r} - \bar{R}_{r1} = 0 \\
& && \text{C3: } \sum_{k=1}^6 q_k(i) - 1 = 0, \forall i \\
& && \text{C4: } q_k(i) - 1 \leq 0, \forall i, k \\
& && \text{C5: } -q_k(i) \leq 0, \forall i, k \\
& && \text{C6: } t(i) - 1 \leq 0, \forall i \\
& && \text{C7: } -t(i) \leq 0, \forall i.
\end{aligned} \tag{14}$$

The Lagrangian function for the above optimization problem is provided in (15) at the top of the next page where $\mu_1, \mu_2, \lambda(i), \alpha_k(i), \beta_k(i), \phi_1(i)$ and $\phi_2(i)$ are the Lagrange multipliers corresponding to constraints C1, C2, C3, C4, C5, C6 and C7, respectively. The KKT conditions include the following:

1) Stationary condition: The differentiation of the Lagrangian function with respect to the primal variables, $q_k(i)$ and $t(i)$ for $\forall i, k$, is equal to zero for the optimal solution, i.e.,

$$\frac{\partial \mathcal{L}}{\partial q_k(i)} = 0 \quad \text{for } \forall i, k \tag{16a}$$

$$\frac{\partial \mathcal{L}}{\partial t(i)} = 0 \quad \text{for } \forall i. \tag{16b}$$

2) Primal feasibility condition: The optimal solution has to satisfy the constraints of the primal problem in (14).

3) Dual feasibility condition: The Lagrange multipliers for the inequality constraints have to be non-negative, i.e.,

$$\alpha_k(i) \geq 0, \forall i, k \tag{17a}$$

$$\beta_k(i) \geq 0, \forall i, k \tag{17b}$$

$$\phi_l(i) \geq 0, \forall i, l \tag{17c}$$

have to hold.

4) Complementary slackness: If an inequality is inactive, i.e., the optimal solution is in the interior of the corresponding set, the corresponding Lagrange multipliers are zero. Thus, we obtain

$$\alpha_k(i)(q_k(i) - 1) = 0, \quad \forall i, k \quad (18a)$$

$$\beta_k(i)q_k(i) = 0, \quad \forall i, k \quad (18b)$$

$$\phi_1(i)(t(i) - 1) = 0, \quad \forall i \quad (18c)$$

$$\phi_0(i)t(i) = 0, \quad \forall i. \quad (18d)$$

A common approach to find a set of primal variables, i.e., $q_k(i), t(i), \forall i, k$ and Lagrange multipliers, i.e., $\mu_1, \mu_2, \lambda(i), \alpha_k(i), \beta_k(i), \phi_l(i), \forall i, k, l$, which satisfy the KKT conditions is to start with the complementarity slackness conditions and see if the inequalities are active or not. Combining these results with the primal feasibility and dual feasibility conditions, we obtain various possibilities. Then, from these possibilities, we obtain one or more candidate solutions from the stationary conditions and the optimal solution is surely one of these candidates. In the following subsections, we first pursue this procedure in order to find optimal $q_k^*(i)$ and $t^*(i)$ for $\forall i, k$, and then, we construct the protocol introduced in Theorem 2 based on three exclusive SNR regions.

A. Optimal $q_k^*(i)$

In order to determine the optimal selection policy, we differentiate the Lagrangian function in (15) with respect to $q_k(i)$, for $k = 1, \dots, 6$, equate the result to zero, and obtain the corresponding $q_k(i)$. This leads to

$$q_1^*(i) : -\frac{1}{N}(1 - \mu_1)C_{1r}(i) + \lambda(i) + \alpha_1(i) - \beta_1(i) = 0 \quad (19a)$$

$$q_2^*(i) : -\frac{1}{N}(1 - \mu_2)C_{2r}(i) + \lambda(i) + \alpha_2(i) - \beta_2(i) = 0 \quad (19b)$$

$$q_3^*(i) : -\frac{1}{N}[(1 - \mu_1)C_{12r}(i) + (1 - \mu_2)C_{21r}(i)] + \lambda(i) + \alpha_3(i) - \beta_3(i) = 0 \quad (19c)$$

$$q_4^*(i) : -\frac{1}{N}\mu_2 C_{r1}(i) + \lambda(i) + \alpha_4(i) - \beta_4(i) = 0 \quad (19d)$$

$$q_5^*(i) : -\frac{1}{N}\mu_1 C_{r2}(i) + \lambda(i) + \alpha_5(i) - \beta_5(i) = 0 \quad (19e)$$

$$q_6^*(i) : -\frac{1}{N}[\mu_1 C_{r2}(i) + \mu_2 C_{r1}(i)] + \lambda(i) + \alpha_6(i) - \beta_6(i) = 0. \quad (19f)$$

Without loss of generality, we obtain the necessary condition for $q_1^*(i) = 1$ and then generalize the result to $q_k^*(i) = 1$ for $k = 2, \dots, 6$. From constraint C3 in (14), the other selection variables are zero, i.e., $q_k^*(i) = 0$, for $k = 2, 3, 4, 5, 6$. Furthermore, from (18a), we obtain $\alpha_k(i) = 0$, for $k = 2, 3, 4, 5, 6$ and from (18b), we obtain $\beta_1(i) = 0$. Then, by substituting these values into (19), we obtain

$$\lambda(i) + \alpha_1(i) = (1 - \mu_1)C_{1r}(i) \triangleq \Lambda_1(i) \quad (20a)$$

$$\lambda(i) - \beta_2(i) = (1 - \mu_2)C_{2r}(i) \triangleq \Lambda_2(i) \quad (20b)$$

$$\lambda(i) - \beta_3(i) = (1 - \mu_1)C_{12r}(i) + (1 - \mu_2)C_{21r}(i) \triangleq \Lambda_3(i) \quad (20c)$$

$$\lambda(i) - \beta_4(i) = \mu_2 C_{r1}(i) \triangleq \Lambda_4(i) \quad (20d)$$

$$\lambda(i) - \beta_5(i) = \mu_1 C_{r2}(i) \triangleq \Lambda_5(i) \quad (20e)$$

$$\lambda(i) - \beta_6(i) = \mu_1 C_{r2}(i) + \mu_2 C_{r1}(i) \triangleq \Lambda_6(i) \quad (20f)$$

where $\Lambda_k(i)$ is referred to as selection metric. By subtracting (20a) from the rest of the equations in (20), we obtain

$$\Lambda_1(i) - \Lambda_k(i) = \alpha_1(i) + \beta_k(i), \quad k = 2, 3, 4, 5, 6. \quad (21)$$

From the dual feasibility conditions given in (17a) and (17b), we have $\alpha_k(i), \beta_k(i) \geq 0$. By inserting $\alpha_k(i), \beta_k(i) \geq 0$ in (21), we obtain the necessary condition for $q_1^*(i) = 1$ as

$$\Lambda_1(i) \geq \max \{\Lambda_2(i), \Lambda_3(i), \Lambda_4(i), \Lambda_5(i), \Lambda_6(i)\}. \quad (22)$$

Repeating the same procedure for $q_k^*(i) = 1$, $k = 2, \dots, 6$, we obtain the necessary condition for selecting transmission mode \mathcal{M}_{k^*} in the i -th time slot as

$$\Lambda_{k^*}(i) \geq \max_{k \in \{1, \dots, 6\}} \{\Lambda_k(i)\}, \quad (23)$$

where the Lagrange multipliers μ_1 and μ_2 are chosen such that C1 and C2 in (14) hold and the optimal value of $t(i)$ in $C_{12r}(i)$ and $C_{21r}(i)$ is obtained in the next subsection. We note that μ_1 and μ_2 are long-term thresholds (Lagrange multipliers) which only depend on the statistics of the channels. Moreover, we prove in Appendix D that the optimal values of μ_1 and μ_2 are in the interval $[0, 1]$. Moreover, for $\mu_1 \neq 0, 1$ and $\mu_2 \neq 0, 1$, the probability that two selection metrics in (20) are equal is zero due to the randomness of the time-continuous channel gains. Therefore, the necessary condition for selecting transmission mode \mathcal{M}_k in (23) is in fact sufficient and is the optimal selection policy. However, if $\mu_1 = 0, 1$ or $\mu_2 = 0, 1$, then some of the selection metrics in (20) are equal for all time slots. In this case, we have the freedom to choose between the modes which have the same value of the selection metric in (20) as long as the long-term constraints C1 and C2 in (14) hold. One way of selecting between the modes for which the selection metrics $\Lambda_k(i)$ are identical is to use a probabilistic approach via coin flips. Therefore, let $X_n(i) \in \{0, 1\}$ be the outcomes of the n -th coin flip in the i -th time slot with the probabilities of the possible outcomes defined as $\Pr\{X_n(i) = 1\} = p_n$ and $\Pr\{X_n(i) = 0\} = 1 - p_n$. In order to include the coin flip in (23), we write the necessary and sufficient condition for the selection of mode \mathcal{M}_k in the i -th time slot as

$$\Lambda_k(i) > \max_{k \in \{1, \dots, 6\}} \{\mathcal{I}_k(i) \Lambda_k(i)\} \quad (24)$$

where $\mathcal{I}_k(i) \in \{0, 1\}$ is a binary indicator variable which is equal to zero for the modes that can not be selected in the i -th time slot and one otherwise. As will be seen, the value of $\mathcal{I}_k(i)$, depending on the average SNRs, is either deterministic or obtained as a function of the outcome of the coin flip(s).

B. Optimal $t^*(i)$

To obtain the optimal $t^*(i)$ for the multiple access mode, we assume $q_3^*(i) = 1$ and calculate the derivative in (16b) which leads to

$$t^*(i) : -\frac{1}{N}(\mu_1 - \mu_2)[C_r(i) - C_{1r}(i) - C_{2r}(i)] + \phi_1(i) - \phi_0(i) = 0. \quad (25)$$

Now, we investigate the following possible cases for $t^*(i)$:

Case 1: If $0 < t^*(i) < 1$, then from (18c) and (18d), we obtain $\phi_l(i) = 0$, for $l = 0, 1$. From (25) and by considering $C_r(i) - C_{1r}(i) - C_{2r}(i) \leq 0$, we also obtain $\mu_1 = \mu_2 = \mu$. Combining these results into (23), leads to

$$\Lambda_3(i) = (1 - \mu)C_r(i) \geq \max\{\Lambda_1(i), \Lambda_2(i)\} \quad (26a)$$

$$\Lambda_6(i) = \mu C_{r1}(i) + \mu C_{r2}(i) \geq \max\{\Lambda_4(i), \Lambda_5(i)\}. \quad (26b)$$

The inequalities in (26a) and (26b) hold with equality with non-zero probability if and only if $\mu = 1$ and $\mu = 0$, respectively, however, this leads to a contradiction as shown in Appendix D. Therefore, only \mathcal{M}_3 and \mathcal{M}_6 are selected for the optimal strategy. Hence, we can use the notation in (24), by setting $\mathcal{I}_k(i) = 0$, $k = 1, 2, 4, 5$ and $\mathcal{I}_k(i) = 1$, $k = 3, 6$, $\forall i$.

In this case, the values of $t(i)$, $\forall i$, and the value of μ are found such that constraints C1 and C2 in (14) hold. Note that when C1 and C2 hold, the following also holds

$$\bar{R}_{1r} + \bar{R}_{2r} = \bar{R}_{r1} + \bar{R}_{r2}. \quad (27)$$

Now, since $\Lambda_3(i)$ and $\Lambda_6(i)$ are the only selection metrics for which $I_k(i) \neq 0$, for $k = 3, 6$, $\forall i$, we observe that both $\Lambda_3(i)$ and $\Lambda_6(i)$, and both the left and right hand side in (27) are independent of the value of $t(i)$, $\forall i$. Thus, we can set μ independently from $t(i)$ such that (27) is satisfied. We note that the optimal μ is unique since increasing μ increases the right hand side of (27) and decreases the left hand side, simultaneously. Since the sum rate does not depend on $t(i)$, i.e., $\bar{R}_{1r} + \bar{R}_{2r} = \frac{1}{N} \sum_{i=1}^N q_3(i)C_r(i)$, any choice of $t(i)$, $\forall i$, that satisfies C1 in (14) is optimal. Then, if C1 in (14) and (27) hold, C2 in (14) also holds. As a result, we have satisfied both C1 and C2 by satisfying C1 in (14) and (27). Therefore, we assume $t(i) = t$, $\forall i$, and t is given by

$$t = \frac{E\{q(C_r - C_{2r})\} - E\{(1 - q)C_{r2}\}}{E\{q(C_r - C_{1r} - C_{2r})\}} \quad (28)$$

where

$$q(i) = \begin{cases} 1, & \text{if } \frac{C_r(i)}{C_{r1}(i) + C_{r2}(i)} \geq \frac{\mu}{\mu - 1} \\ 0, & \text{otherwise} \end{cases}. \quad (29)$$

We note that the necessary condition for the optimality of this case is that $0 < t < 1$. For the cases which lead to $t \leq 0$ or $t \geq 1$, we cannot have $\mu_1 = \mu_2$, and therefore, we obtain either $t(i) = 1$ or $t(i) = 0$. In these cases, we need different selection policies which are investigated in the following.

Case 2: If $t(i) = 0$, then from (18c), we obtain $\phi_1(i) = 0$ and from (17c), we obtain $\phi_0(i) \geq 0$. Combining these results with (25), we obtain $\mu_1 \geq \mu_2$. Then, by substituting these results in (20), we obtain

$$\begin{aligned}\Lambda_3(i) &= (1 - \mu_1)[C_r(i) - C_{2r}(i)] + (1 - \mu_2)C_{2r}(i) \\ &= (1 - \mu_1)C_r(i) + (1 - \mu_2)C_{2r}(i) \\ &\geq \max\{\Lambda_1(i), \Lambda_2(i)\}\end{aligned}\tag{30a}$$

$$\Lambda_6(i) = \mu_2 C_{r1}(i) + \mu_1 C_{r2}(i) \geq \max\{\Lambda_4(i), \Lambda_5(i)\}.\tag{30b}$$

The expressions in (30a) and (30b) hold with equality with non-zero probability if and only if $\mu_1 = 1$ and $\mu_2 = 0$, respectively, otherwise only the inequality holds, respectively. Therefore, we consider the following four possibilities for the thresholds μ_1 and μ_2 .

a) If $0 < \mu_1 < 1$ and $0 < \mu_2 < 1$, then, (30) holds with inequality and therefore, only \mathcal{M}_3 and \mathcal{M}_6 are selected. Hence, for this case, we can set $\mathcal{I}_k(i) = 0$, $k = 1, 2, 4, 5$ and $\mathcal{I}_k(i) = 1$, $k = 3, 6$ for $\forall i$. Since by simultaneously increasing μ_1 and μ_2 , the left hand side of (27) decreases and the right hand side increases, we can always find a pair (μ_1, μ_2) which satisfies this constraint. Therefore, the necessary condition for the optimality in this case is that among the candidates (μ_1, μ_2) that satisfy (27), there exists a point which satisfies either C1 or C2 in (14) (we note that since we assume (27) holds, if C1 holds, C2 must hold too and vice versa). Moreover, if there exists such a point then it is unique. To prove this, note that the candidates (μ_1, μ_2) that satisfy (27) form a monotonically decreasing curve in the plain of (μ_1, μ_2) , since by increasing both μ_1 and μ_2 , we simultaneously decrease the left hand side and increase the right hand side of (27). Moreover, increasing μ_1 and decreasing μ_2 results in decreasing \bar{R}_{1r} and \bar{R}_{r1} , and increasing \bar{R}_{2r} and \bar{R}_{r2} . Therefore, there exists a unique (μ_1, μ_2) which satisfies C1 or C2 in (14), otherwise, either μ_1 or μ_2 reaches a boundary value, i.e., zero or one, which is discussed in the following.

b) If $\mu_1 = 1$ and $0 < \mu_2 < 1$, then from (20), we obtain

$$\Lambda_2(i) = \Lambda_3(i) = (1 - \mu_2)C_{2r}(i) \geq \Lambda_1(i)\tag{31a}$$

$$\Lambda_6(i) = \mu_2 C_{r1}(i) + C_{r2}(i) \geq \max\{\Lambda_4(i), \Lambda_5(i)\}.\tag{31b}$$

The probability that the above expressions hold with equality is zero. Therefore, we can set $\mathcal{I}_k(i) = 0$, $k = 1, 4, 5$ and $\mathcal{I}_6(i) = 1$, $\forall i$. Moreover, we observe that constraint C2 in (14), i.e., $\bar{R}_{2r} = \bar{R}_{r1}$, only depends on μ_2 , and as μ_2 decreases, \bar{R}_{2r} increases and \bar{R}_{r1} decreases. Since for $\mu_2 = 0$ we obtain $\bar{R}_{1r} = 0 < \bar{R}_{r2}$, a sufficient and necessary condition for the existence of a μ_2 such that constraint C2 in (14) holds is $P_2 > P_r$. Since both thresholds are obtained, we could determine when to select mode \mathcal{M}_6 or $\{\mathcal{M}_2, \mathcal{M}_3\}$. Next, we need to determine the optimal policy to select between \mathcal{M}_2 and \mathcal{M}_3 if for some channel realizations, we obtain $\Lambda_2(i) = \Lambda_3(i) > \Lambda_6(i)$ such that constraint C1 in (14) holds and the maximum sum rate is achieved. Since the maximum sum rate is fixed to $E\{q_6(C_{r1} + C_{r2})\}$ and it is only a function of μ_2 , different strategies of satisfying C1 in (14) yield the same sum rate. Thus, we consider a simple probabilistic approach for choosing \mathcal{M}_3 with probability p_1 and \mathcal{M}_2 with

probability $1 - p_1$. To implement the probabilistic strategy, we set $\mathcal{I}_3(i) = 1 - \mathcal{I}_2(i) = X_1(i)$ where $X_1(i)$ is the outcome of a coin flip with probability $p_1 = \frac{E\{q_6 C_{r2}\}}{E\{(1-q_6)[C_r - C_{2r}]\}}$ where p_1 is chosen such that constraint C1 in (14) is satisfied.

c) If $0 < \mu_1 < 1$ and $\mu_2 = 0$, then from (20), we obtain

$$\begin{aligned} \Lambda_3(i) &= (1 - \mu_1)[C_r(i) - C_{2r}(i)] + C_{2r}(i) \\ &\geq \max\{\Lambda_1(i), \Lambda_2(i)\} \end{aligned} \quad (32a)$$

$$\Lambda_5(i) = \Lambda_6(i) = \mu_1 C_{r2}(i) \geq \Lambda_4(i). \quad (32b)$$

Similar to Case b, the probability that the above expressions hold with equality is zero. Therefore, we set $\mathcal{I}_k(i) = 0$, $k = 1, 2, 4$, $\mathcal{I}_3(i) = 1$, and $\mathcal{I}_6(i) = 1 - \mathcal{I}_5(i) = X_2(i)$ with $p_2 = \frac{E\{q_3 C_{2r}\}}{E\{(1-q_3)C_{r1}\}}$ for $\forall i$. Moreover, the necessary condition for this case is that $P_2 < P_r$ and the sum rate is $E\{q_3 C_r\}$.

d) If $\mu_1 = 1$ and $\mu_2 = 0$, then from (20), we obtain

$$\Lambda_2(i) = \Lambda_3(i) = C_{2r}(i) \geq \Lambda_1(i) \quad (33a)$$

$$\Lambda_5(i) = \Lambda_6(i) = C_{r2}(i) \geq \Lambda_4(i). \quad (33b)$$

The probability that the above expressions hold with equality is zero and therefore, we can set $\mathcal{I}_k(i) = 0$, $k = 1, 4$. Moreover, one can easily conclude that the necessary condition for this case is $P_2 = P_r$, otherwise, only one of $\{\mathcal{M}_2, \mathcal{M}_3\}$ or $\{\mathcal{M}_5, \mathcal{M}_6\}$ is selected in all time slots which violates constraints C1 and C2 in (14). Thus, we obtain $\Lambda_2(i) = \Lambda_3(i) = \Lambda_5(i) = \Lambda_6(i)$, $\forall i$. Moreover, considering that C1 and C2 in (14) hold, the sum rate is $\bar{R}_{2r} + \bar{R}_{r2} = E\{(q_2 + q_3)C_{2r}\} + E\{(q_5 + q_6)C_{r2}\} = E\{C_{2r}\} = E\{C_{r2}\}$. Moreover, the maximum sum rate is achieved by a probabilistic approach as follows

$$\begin{cases} \mathcal{I}_2(i) = X_3(i)[1 - X_4(i)] \\ \mathcal{I}_3(i) = X_3(i)X_4(i) \\ \mathcal{I}_5(i) = [1 - X_3(i)][1 - X_5(i)] \\ \mathcal{I}_6(i) = [1 - X_3(i)]X_5(i) \end{cases} \quad (34)$$

where the coin flip probabilities $p_4 = \frac{1-p_3}{p_3} \frac{E\{C_{r2}\}}{E\{C_r - C_{2r}\}}$ and $p_5 = \frac{p_3}{1-p_3} \frac{E\{C_{2r}\}}{E\{C_{r1}\}}$ are chosen such that constraints C1 or C2 in (14) are satisfied, respectively, and probability p_3 is chosen to achieve the optimal sum rate. However, for the assumed values of p_4 and p_5 , the sum rate is $\bar{R}_{12} + \bar{R}_{21} = E\{C_{2r}\} = E\{C_{r2}\}$ which is indeed the maximum sum rate. Therefore, any value of p_3 which results in an acceptable value of p_4 and p_5 , i.e., $0 \leq p_4, p_5 \leq 1$, is optimal. From this condition, we obtain $\frac{E\{C_{r2}\}}{E\{C_r\}} \leq p_3 \leq \frac{E\{C_{r1}\}}{E\{C_{r1} + C_{2r}\}}$, where $p_3 = \frac{E\{C_{r2}\}}{E\{C_r\}}$ and $p_3 = \frac{E\{C_{r1}\}}{E\{C_{r1} + C_{2r}\}}$ lead to $p_4 = 1$ and $p_5 = 1$, respectively.

Case 3: If $t(i) = 1$, then considering (18d), we obtain $\phi_0(i) = 0$ and from (17c), we obtain $\phi_1(i) \geq 0$. Combining these results with (25), we obtained $\mu_1 \leq \mu_2$. Following a similar procedure as for Case 2, we obtain similar results.

C. SNR Regions

We obtained different selection policies and the necessary conditions of optimality for each of them. Now, we determine which of the derived selection policies will be used based on the statistics of the channels and the powers of the nodes. To this end, we define three mutually exclusive SNR regions \mathcal{R}_1 , \mathcal{R}_2 , and \mathcal{R}_0 . Each of the necessary conditions is realized only in one of these SNR regions and since the SNR regions are mutually exclusive and the necessary conditions are mutually exclusive too, the selection policy for each necessary condition is optimal for the corresponding SNR region.

First, we observe that the multiple access and broadcast modes are selected with non-zero probability in all the discussed cases. However, the point-to-point modes are selected only when thresholds μ_1 and μ_2 have a boundary value, i.e., zero or one. By this observation, we define the SNR regions \mathcal{R}_1 , \mathcal{R}_2 and \mathcal{R}_0 based on the utilization of point-to-point modes as follows:

SNR Region \mathcal{R}_0 : In this SNR region, none of the point-to-point modes are selected. Moreover, the necessary conditions for the thresholds are $0 < \mu_1 < 1$ and $0 < \mu_2 < 1$.

SNR Region \mathcal{R}_1 : In this SNR region, one or both of point-to-point modes \mathcal{M}_2 and \mathcal{M}_5 are selected. Moreover, the necessary condition for the thresholds is $\mu_1 > \mu_2$ where at least one of the thresholds has a boundary value. Specifically, in this SNR region, if $P_2 > P_r$, we obtain $\mu_1 = 1$ and $0 < \mu_2 < 1$ and mode \mathcal{M}_2 is selected in some time slots, whereas if $P_2 < P_r$, we obtain $0 < \mu_1 < 1$ and $\mu_2 = 0$ and mode \mathcal{M}_5 is selected in some time slots, and finally if $P_2 = P_r$, we obtain $\mu_1 = 1$ and $\mu_2 = 0$ and both modes \mathcal{M}_2 and \mathcal{M}_5 can be selected in some time slots.

SNR Region \mathcal{R}_2 : In this SNR region, one or both of the point-to-point modes \mathcal{M}_1 and \mathcal{M}_4 is selected. Moreover, the necessary condition for the thresholds is $\mu_1 < \mu_2$ where at least one of the thresholds has a boundary value. Similar to SNR region \mathcal{R}_1 , in this SNR region, if $P_1 > P_r$, we obtain $0 < \mu_1 < 1$ and $\mu_2 = 1$ and mode \mathcal{M}_1 is selected in some time slots, whereas if $P_1 < P_r$, we obtain $\mu_1 = 0$ and $0 < \mu_2 < 1$ and mode \mathcal{M}_4 is selected in some time slots, and finally if $P_1 = P_r$, we obtain $\mu_1 = 0$ and $\mu_2 = 1$ and both modes \mathcal{M}_1 and \mathcal{M}_4 can be selected in some time slots.

Roughly speaking, in SNR region \mathcal{R}_1 , the user 1-to-relay link is much stronger than the user 2-to-relay link, and one or both of the point-to-point modes between the relay and user 2 are selected. Similarly, in SNR region \mathcal{R}_2 , the user 2-to-relay link is much stronger than the user 1-to-relay link, and one or both of the point-to-point modes between the relay and user 1 are selected. However, in SNR \mathcal{R}_0 , the quality of user 1-to-relay link is similar to that of the user 2-to-relay link, and none of the point-to-point modes is selected. To easily distinguish the SNR regions based on the channel statistics, we investigate the transition conditions between the regions. We note that as the quality of the user 1-to-relay link improves compared to the user 2-to-relay link, the SNR region changes from \mathcal{R}_2 to \mathcal{R}_0 and then from \mathcal{R}_0 to \mathcal{R}_1 . Therefore, the transition condition between SNR regions \mathcal{R}_1 and \mathcal{R}_0 occurs when the coin flip probabilities are equal to one, i.e., $p_1, p_2, p_4, p_5 \overset{\mathcal{R}_0}{\underset{\mathcal{R}_1}{\gtrless}} 1$. Then, based on whether P_r is larger, smaller or equal to P_2 , we use the optimal strategy introduced in Case b, c, and d, respectively. Therefore, the SNR region is

\mathcal{R}_1 if and only if

$$\begin{aligned} & \{P_2 > P_r \text{ AND } \omega_1 < 1\} \\ & \text{OR } \{P_2 < P_r \text{ AND } \omega_2 < 1\} \\ & \text{OR } \{P_2 = P_r \text{ AND } \omega_3^l < \omega_3^u\} \end{aligned} \quad (35)$$

where $\omega_1 = \frac{E\{qC_{r2}\}}{E\{(1-q)[C_r - C_{2r}]\}}$, $\omega_2 = \frac{E\{qC_{2r}\}}{E\{(1-q)C_{r1}\}}$, $\omega_3^l = \frac{E\{C_{r2}\}}{E\{C_r\}}$, and $\omega_3^u = \frac{E\{C_{r1}\}}{E\{C_{r1} + C_{2r}\}}$ and $q(i)$ is defined as

$$q(i) = \begin{cases} 1, & \text{if } \left\{ P_2 > P_r \text{ AND } [\Lambda_3 \leq \Lambda_6]_{\mu_1=1, \mu_2=\mu_2^*}^{t(i)=0, \forall i} \right\} \\ & \text{OR } \left\{ P_2 < P_r \text{ AND } [\Lambda_3 \geq \Lambda_6]_{\mu_1=\mu_1^*, \mu_2=0}^{t(i)=0, \forall i} \right\} \\ 0, & \text{otherwise} \end{cases} \quad (36)$$

where if $P_2 > P_r$, the threshold μ_2^* is chosen such that $E\{(1-q)C_{2r}\} = E\{qC_{r1}\}$ holds, whereas, if $P_2 < P_r$, the threshold μ_1^* is chosen such that $E\{q[C_r - C_{2r}]\} = E\{(1-q)C_{r2}\}$ holds. In a similar manner, one can distinguish between SNR region \mathcal{R}_2 and \mathcal{R}_0 . We note that all conditions of optimality for these three exclusive SNR regions are obtained in this section and the thresholds μ_1 and μ_2 which resulted in the maximum sum rate are shown to be unique. Thus the proposed solution is optimal. This completes the proof.

APPENDIX C

PROOF OF BINARY RELAXATION

In this appendix, we prove that the optimal solution of the problem with the relaxed constraints, $0 \leq q_k(i) \leq 1$, can also be achieved if the values of $q_k(i)$ are binary, i.e., if $q_k(i) \in \{0, 1\}$. Therefore, the binary relaxation does not change the maximum sum rate. We start with noting that if one of the $q_k(i)$, $k = 1, \dots, 6$, adopts a non-binary value in the optimal solution, then in order for constraint C3 in (4) to be satisfied, there has to be at least one other non-binary selection variable in that time slot. Assuming that the mode indices of the non-binary selection variables are k' and k'' in the i -th time slot, we obtain $\alpha_k(i) = 0$, $k = 1, \dots, 6$ from (18a), $\beta_{k'}(i) = 0$ and $\beta_{k''}(i) = 0$ from (18b). Then, by substituting these values into (19), we obtain

$$\lambda(i) = \Lambda_{k'}(i) \quad (37a)$$

$$\lambda(i) = \Lambda_{k''}(i) \quad (37b)$$

$$\lambda(i) - \beta_3(i) = \Lambda_k(i), \quad k \neq k', k''. \quad (37c)$$

From (37a) and (37b), we obtain $\Lambda_{k'}(i) = \Lambda_{k''}(i)$ and by subtracting (37a) and (37b) from (37c), we obtain

$$\Lambda_{k'}(i) - \Lambda_k(i) = \beta_k(i), \quad k \neq k', k'' \quad (38a)$$

$$\Lambda_{k''}(i) - \Lambda_k(i) = \beta_k(i), \quad k \neq k', k''. \quad (38b)$$

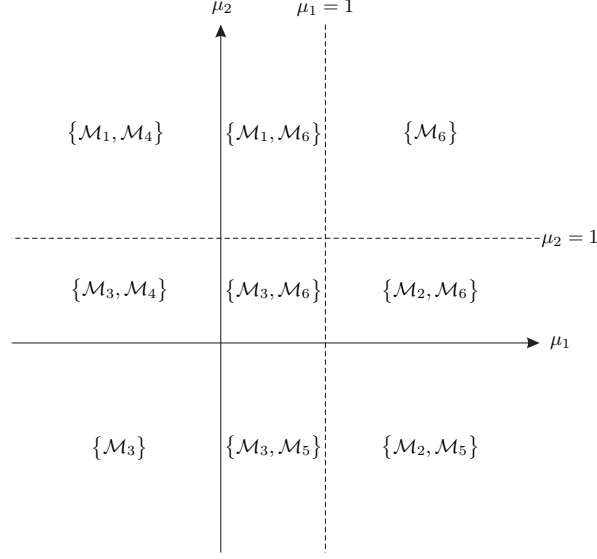


Fig. 4. Modes with non-negative powers in the space of (μ_1, μ_2) .

From the dual feasibility condition given in (17b), we have $\beta_k(i) \geq 0$ which leads to $\Lambda_{k'}(i) = \Lambda_{k''}(i) \geq \Lambda_k(i)$. However, as a result of the randomness of the time-continuous channel gains, $\Pr\{\Lambda_{k'}(i) = \Lambda_{k''}(i)\} > 0$ holds for some transmission modes $\mathcal{M}_{k'}$ and $\mathcal{M}_{k''}$ if and only if the optimal value of either μ_1 or μ_2 is at a boundary, i.e., $\mu_1 = 0, 1$ or $\mu_2 = 0, 1$. In these cases, we observe that $\Lambda_{k'}(i) = \Lambda_{k''}(i)$, $\forall i$, and we have shown in Appendix B that for $\mu_1 = 0, 1$ and $\mu_2 = 0, 1$, we can obtain the optimal sum rate by using a probabilistic approach to choose $\mathcal{M}_{k'}$ with probability p and $\mathcal{M}_{k''}$ with probability $1 - p$, which results in a binary value for the selection variables $q_{k'}(i)$ and $q_{k''}(i)$. This completes the proof.

APPENDIX D THRESHOLD REGIONS

In this appendix, we determine the interval which contains the optimal values of μ_1, μ_2 . Fig. 4 represents the set of candidate selection modes for all channel realizations in the space of (μ_1, μ_2) . These candidate modes are obtained based on the necessary selection condition introduced in (23) as:

- 1) If $\mu_1 < 1$ and $\mu_2 > 1$, then \mathcal{M}_1 is a candidate for selection since we obtain $\Lambda_1(i) \geq \max\{\Lambda_2(i), \Lambda_3(i)\}$ from (20).
- 2) If $\mu_1 > 1$ and $\mu_2 < 1$, then \mathcal{M}_2 is a candidate for selection since we obtain $\Lambda_2(i) \geq \max\{\Lambda_1(i), \Lambda_3(i)\}$ from (20).
- 3) If $\mu_1 < 1$ and $\mu_2 < 1$, then, based on (26a) and (30a), \mathcal{M}_3 is a candidate for selection since we obtain $\Lambda_3(i) \geq \max\{\Lambda_1(i), \Lambda_2(i)\}$ from (20).
- 4) If $\mu_1 < 0$ and $\mu_2 > 0$, then \mathcal{M}_4 is a candidate for selection since we obtain $\Lambda_4(i) \geq \max\{\Lambda_5(i), \Lambda_6(i)\}$ from (20).

5) If $\mu_1 > 0$ and $\mu_2 < 0$, then \mathcal{M}_5 is a candidate for selection since we obtain $\Lambda_5(i) \geq \max\{\Lambda_4(i), \Lambda_6(i)\}$ from (20).

6) If $\mu_1 > 0$ and $\mu_2 > 0$, then \mathcal{M}_6 is a candidate for selection since we obtain $\Lambda_6(i) \geq \max\{\Lambda_4(i), \Lambda_5(i)\}$ from (20).

It is straightforward to see that all sets in Fig. 4 contradict one or both constraints C1 and C2 in (4), except $\{\mathcal{M}_3, \mathcal{M}_6\}$, which corresponds to $0 \leq \mu_1 \leq 1$ and $0 \leq \mu_2 \leq 1$. Note that if μ_1 and μ_2 are at the boundaries, i.e., $\mu_1 = 0, 1$ and $\mu_2 = 0, 1$, then the neighbor modes are also candidates for selection.

We note that μ_1 and μ_2 cannot be simultaneously equal to zero since this results in $\Lambda_4(i) = \Lambda_5(i) = \Lambda_6(i) = 0$, $\forall i$, and thus only the transmission from the users to the relay will be adopted and this contradicts constraints C1 and C2 in (4). Similarly, μ_1 and μ_2 cannot be simultaneously equal to one since it also contradicts constraints C1 and C2 in (4).

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